

①

$$y = -(x-4)(x+2)$$

$$= -(x^2 + 2x - 4x - 8)$$

$$y = -x^2 + 2x + 8$$

$$\frac{dy}{dx} = -2x + 2$$

$$\frac{d^2y}{dx^2} = -2 \quad [\text{maxima when negative}]$$

Maximum point

$$-2x + 2 = 0$$

$$x = 1$$

$$y = -1^2 + 2 + 8$$

$$y = 9$$

point $(1, 9)$

② $f(x) = 2x^2 - 8x + 1$

$$f'(x) = 4x - 8 = 0 \quad \text{at turning point}$$

$$f''(x) = 4 \quad (\text{minimum point})$$

$$4x - 8 = 0$$

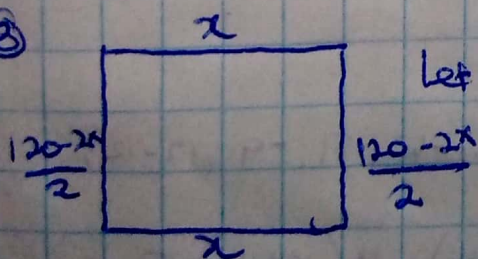
$$x = 2$$

$$y = 2(2^2) - 8(2) + 1$$

$$y = 8 - 16 + 1 = -7$$

min point $(2, -7)$

③

Let length $= x$

$$60 - x + 60 - x + x + x = 120$$

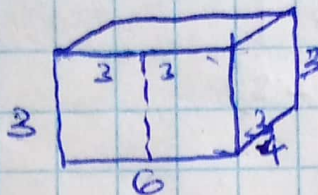
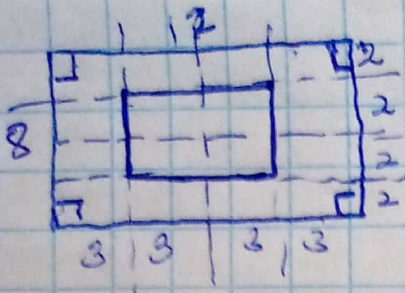
$$\text{Area} = x(60 - x) = 60x - x^2$$

$$\frac{dA}{dx} = 60 - 2x = 0 \quad \text{for Max area}$$

$$60 - 2x = 0$$

$$2x = 60 \quad x = 30 \text{ inch}$$

④



Volume = $A \times h$

$$\frac{dv}{da} = \frac{1}{2} \times 8 = 4$$

$$\text{width} = \frac{1}{2} \times 8 = 4$$

$$\text{volume} = 4 \times 3 \times 6 = 72 \text{ inches}^3$$

① $8x^3 - 4x^2 + 10x - 20$

roots of 20

$(-1, 1, 2, -2, 4, -4, 5, -5, 10, -10, 20, -20)$

$$-1 (-8 - 4 + 10 - 20) \neq 0$$

$$1 (8 - 4 + 10 - 20) \neq 0$$

$$2 (64 - 16 + 20 - 20) \neq 0$$

$$-2 (-64 - 16 - 20 - 20) \neq 0$$

$$4 (8(64) - 4(16) + 10(4) - 20)$$

$$8x^3 - 4x^2 + 10x - 20$$

$$2x^2(x-2) + 10(x-2) = 0$$

$$(2x^2 + 10)(x-2) = 0$$

$$x^2 = -\frac{10}{2}$$

$$x = \sqrt{-5} = \text{Complex root}$$

$$x = 2 \text{ real root.}$$

② $x^3 + 4x^2 - 9x - 36 = 0$

factors of 36 : $-1, 1, 2, -2, 3, -3, 4, -4, 6, -6, 9, -9, 12, -12, 18, -18, 36, -36$

Trial and error

roots 3

factor $(x-3)$

-3

$(x+3)$

-4

$(x+4)$

$$x^2(x+4) - 9(x+4) = 0$$

$$(x^2 - 9)(x+4) = 0$$

$$(x^2 - 9) = 0$$

$$x = \pm 3 \quad x = -4$$

$$x^3 + 4x^2 - 9x - 36 = (x+3)(x-3)(x+4) = 0$$

roots = 3, -3, -4

③ $x^4 - 8x^2 + 12 = 0$

factors of 12 : -1, 1, -2, 2, 3, -3, 4, -4, 6, -6, 12, -12

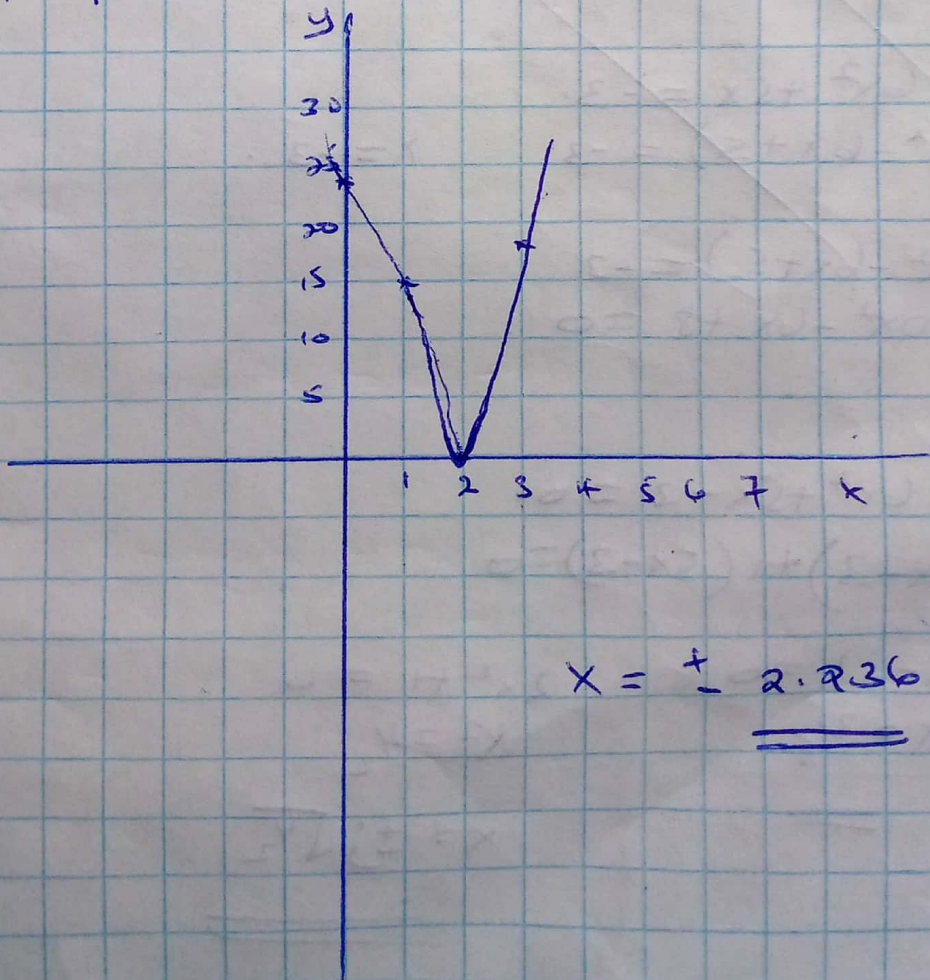
$$x^4 - 8x^2 + 12 = (x^2 - 2)(x^2 - 6)$$

$$x = \pm\sqrt{2}$$

$$x = \pm\sqrt{6}$$

④ $x^4 - 10x^2 + 24$

x	0	1	2	3
y	24	15	0	18



$$x = \pm \underline{\underline{2.236}}$$

$$x^4 - 10x^2 + 24 = (x+2)(x-2)(x+\sqrt{6})(x-\sqrt{6})$$

$$\text{roots} = 2, -2, \sqrt{6}, -\sqrt{6}$$

⑤

$$12x^4 - 16x^2 + 9 = 0.$$

Factors of 9 : 1, -1, 3, -3, 9, -9

$$12x^4 - 16x^2 = -9.$$

$$x^2(12x^2 - 16) = -9.$$

$$x^2 = -9.$$

$$x = \pm j\sqrt{9} = \pm 3j$$

$$12x^2 - 16 = -9.$$

$$12x^2 = -9 + 16$$

$$12x^2 = 7.$$

$$x^2 = 7/12$$

$$x = \pm \sqrt{7/12}.$$

$$\textcircled{e} \quad 10x^3 - 6x^2 + 5x - 2 = 0$$

$$2x^2(5x-3) + 1(5x-3) = 0$$

$$(5x-3) = 0$$

$$x = 3/5$$

$$2x^2 + 1 = 0$$

$$x^2 = -1/2$$

$$x = \pm j\sqrt{1/2}$$